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NEW INSIGHT INTO CALCULATING DISCOUNTS FOR LACK OF MARKETABILITY

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It is often the case that the discount for lack of marketability ("DLOM") is the second largest number determined by appraisers in their business valuations. Yet, DLOM is perhaps the single most misunderstood concept in the business valuation community and perhaps the most subjective number in valuation reports.

WHAT ARE WE TRYING TO MEASURE?

Liquidity represents the ability to sell an investment quickly when the investor decides to sell. Conversely, then, ***lack*** of liquidity represents the cost of failing to realize gains or failing to avoid losses on an investment during the period in which the investor is offering it for sale. With that understanding, DLOM should reflect the volatility of the value of the investment during the period of time that it is being marketed.

But where should we start in our analysis of DLOM? The obvious answer is with publicly traded stocks, because the estimates of rates of return that we use in our valuations come from analysis of publicly traded companies. However, the business valuation community has been misled by an improper presentation of the relative levels of value present in the business equity investment marketplace. The traditional view is –

Strategic Value

Difference reflects synergies

Control Value

Difference reflects the value of control

Publicly Traded Value

Difference reflects the value of marketability

Non-Marketable Minority Value

Some authors recently have been suggesting a modified view wherein control value and publicly traded value may be very close to the same. Nevertheless, it is my opinion that the traditional and modified views of levels of value are incorrect. On a condensed basis, levels of business value should be viewed this way –

Publicly Traded Value

Difference reflects the economic risk of lack of marketability

Non-Marketable Control Value

Difference reflects the economic risk of lack of control

Non-Marketable Minority Value

The basis of my opinion is straightforward. First, the investment returns of publicly traded companies should be viewed as "public company returns" not as "marketable minority returns." For well run companies that are operating optimally for their shareholders, there should be no economic difference between public company operating results and operating results to controlling interests of otherwise identical private companies – the material perquisites of control have been squeezed out of the public companies. If this were not essentially true, then publicly traded companies would not be able to attract capital in the form of fractional ownership. And, in fact, poorly run companies (i.e. those not operating optimally for their shareholders) have difficulty maintaining shareholder value and raising new capital.

Second, strategic value does not enter into the DLOM equation. There are well run publicly traded companies and well run privately held companies. There are also poorly run companies of both types. When a public company is acquired at a premium above its publicly traded value it is a reflection of the perception that the acquired company is not maximizing its economic opportunities and shareholder value. Although strategic opportunity suggests a value to being in control, it does not suggest that control is worth more than liquidity. Well-run publicly traded companies (i.e. those that are maximizing their economic opportunities and shareholder value) are not taken private – they are too expensive. For these reasons, I believe that the so-called "control premium studies" are misused when they are used to suggest that control is worth more than liquidity.

Third, when comparing the value drivers of well run publicly traded and well run privately controlled businesses, we find that the only real difference is liquidity or its lack:

Public Companies	**Private Companies**
Earnings / Cash Flow	Earnings / Cash Flow
Growth	Growth
Industry Risk	Industry Risk
Size Risk	Size Risk
Market Fluctuations	Market Fluctuations
Liquidity	**No Liquidity**

Consider these thoughts: (1) Risk adjusted rates of return are fungible.[1] (2) There is a transaction cost to becoming and continuing as a publicly traded company. This creates a

[1] See Eric W. Nath, ASA, and M. Mark Lee, CFA "Acquisition Premium High Jinks," 2003 International Appraisal Conference, American Society of Appraisers; Eric W. Nath, ASA, "How Public Guideline Companies Represent 'Control' Value for a Private Company," Business Valuation Review, Vol. 16, No. 4, December 1997; and Eric W. Nath, "Control Premiums and Minority Discounts in Private Companies," Business Valuation Review, Vol. 9, No. 2, June 1990.

disincentive that can only be justified by (a) greater access to capital, and (b) the "pop" in value that the pre-IPO owners receive when their business goes public. (3) If control were worth more than liquidity, then the owners of privately held businesses would have a further disincentive to going public. (4) If control were more valuable than liquidity, then there would be no public companies.[2] (5) If control were worth more than liquidity, then large private equity firms such as Blackstone and KKR would not be converting to publicly traded companies. Thus it seems counter-intuitive that control should be viewed as equal in value to – or even more valuable than – liquidity.

Under otherwise identical circumstances, any given investment should have a greater value if it is immediately marketable than if it is not. Why is this so? Because liquidity allows the investor to avoid the economic risks of illiquidity. The notion of a control premium vis-à-vis public company values is illogical. Such premiums mathematically equate to lower rates of return. But since it is expected that it would take longer to sell a controlling interest in an optimally run private company than an interest in an otherwise identical public company, the required rate of return of the private company investor should be greater, not lower, than that of the public company investor. Thus, private company values should reflect a discount, not a premium, relative to comparable public company values.

[2] Id.

Figure 1

ALTERNATIVE VIEW OF THE LEVELS OF BUSINESS VALUE

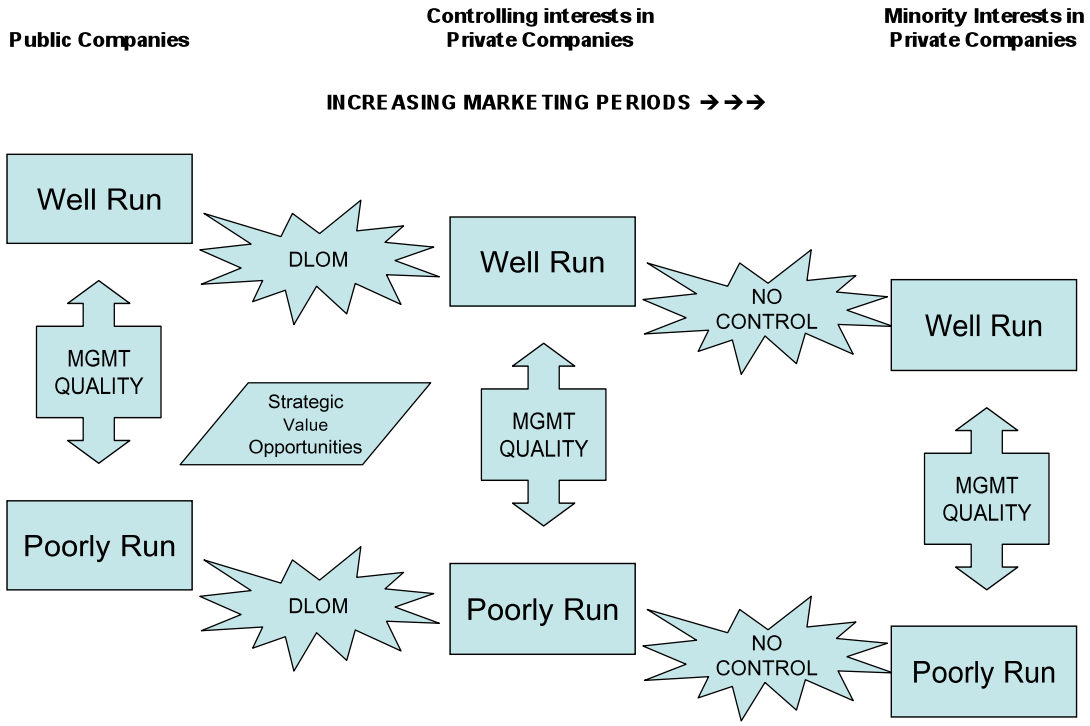


Figure 1 presents my alternative view of the relative levels of business investment value in greater dimension. The depiction shows how well run and poorly run private companies relate to each other and how the opportunity to realize strategic value (including synergy) arises from the conversion of poorly run firms into firms that hopefully will be well run. The depiction also demonstrates that all privately held companies – even controlling interests – are subject to DLOM.[3] Even assuming all other things being equal, it simply takes longer to sell a controlling interest in a privately held business than it takes to sell an interest in a publicly traded company. Minority interests in privately held companies are worth less than controlling interests for two reasons: (1) such minorities generally lack the ability of controlling owners have to realize the perquisites of ownership and (2) the economic risks of lack of control result in longer periods of time to sell minority interests than it takes to sell the controlling interest in the same private company.

[3] It has been suggested by some practitioners that controlling interests are not subject to a discount for lack of marketability because the earnings and cash flow of the company offset the discount while it is being held for sale. This argument fails because (1) it relies on a flawed view of the levels of value that ignores the facts that (a) rates of return derive from analysis of publicly traded stocks, and (b) liquidity is the only driver of value of publicly traded companies not present in privately held companies; (2) the economic circumstance of holding period earnings and cash flow also exists for minority interests; and (3) the holding period earnings and cash flow of both controlling interest and minority interest investments are necessarily already included in the capitalized or discounted values of the investments.

HOW THE EMPIRICAL STUDIES OF DISCOUNTS AND LIQUIDITY RELATE TO EACH OTHER

Conventional business valuation has used the well-publicized results of restricted stock studies, pre-IPO studies, and registered versus unregistered stock studies to effectively guess at appropriate DLOM percentages to use in their valuation reports. Understandably, such subjective means of applying the traditional approaches have been broadly unsatisfactory to the valuation community and the courts.

Figure 2

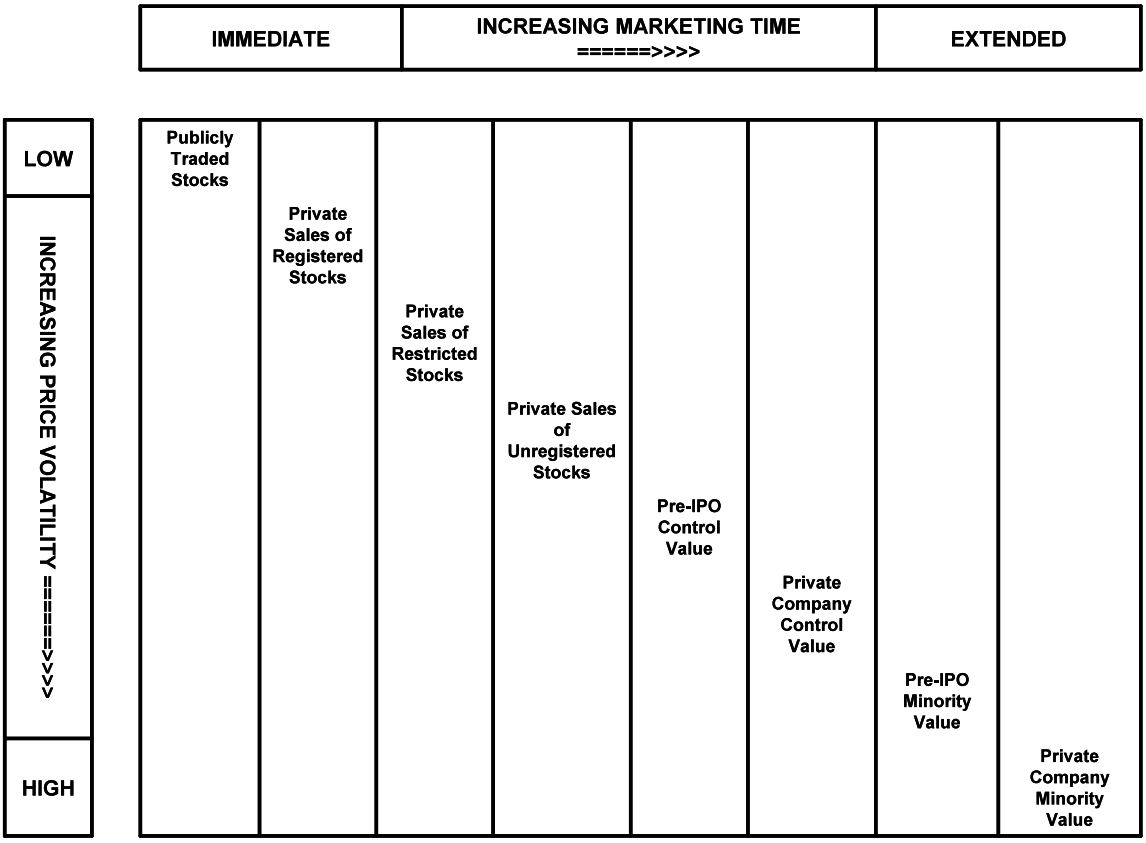


Figure 2 presents a relational stratification of the types of empirical studies that researchers have performed to explore the cost of illiquidity. I have attempted to present the studies in relative position based on marketing time and volatility assuming all other aspects of investment were equal. The presentation is instructive in enhancing understanding of what the various studies are measuring, how they relate to publicly traded values, and the extent to which they meet the needs of business valuation.

- Publicly traded companies are the standard against which all of the studies measure results and from which rates of return are calculated. Interests in publicly traded companies are worth more than interests in identical privately

held companies because they can be sold immediately to realize gains and to avoid losses. Interests in privately held companies cannot.

- Private sales of publicly registered stocks typically involve large blocks of stock that could be sold into the public marketplace, but which would materially adversely affect stock prices if the entire block were to be dumped into the market at once. Avoiding that effect results in an extended period of time to liquidate the investment position in the public market during which time the investor is subject to market risk. Negotiating a private sale of the block can accelerate liquidating the position, but the need to find a buyer with the wherewithal to purchase the block restricts the number of potential buyers and represents a diminution of demand for the stock. Furthermore, although private sales of large blocks of registered stocks can somewhat mitigate the market risk, the risk does not go away. The buyer of the block assumes the risks, in turn, of having to sell into a limited pool of buyers or slowly feeding the block into the public market. These risks require compensation by means of a discount (i.e. DLOM).
- Private sales of restricted stocks in public companies have the same price risks as private sales of large blocks of registered stocks, but have the additional risk of being locked out of the public market for specific periods of time or being subject to restrictive "dribble out" rules. Accordingly, restricted stocks often can only be sold quickly in private sale transactions, which take longer than it does to sell unrestricted stocks in the public market.[4] The result is that a restricted registered stock is worth less than an unrestricted stock in the same company because of the greater market risk associated with the extended marketing period.
- Private sales of unregistered stocks in public companies typically involve large blocks of stock. They are worth less than equivalent blocks of registered stock (whether restricted or unrestricted) in the same publicly traded company because there is a cost to ultimate registration of the stock that further restricts the potential number of buyers of the block.[5] This results in relatively greater

[4] Some restricted stocks cannot be sold at all for contractually determined periods of time. Such investments have even greater economic risks than those merely subject to the "dribble out" rules.

[5] This discount is considered by Mukesh Bajaj, David J. Dennis, Stephen P. Ferris and Atulya Sarin in their paper "Firm Value and Marketability Discounts." Their study isolates the value of liquidity by comparing the stock sales of 88 companies that had sold both registered and unregistered stock private offerings. This approach does not, however, address the discount applicable to the additional time it takes to sell controlling or minority interests in private companies. Instead, it measures the value of stock registration. See Section IV.C of "Firm Value and Marketability Discounts."

uncertainty, a relatively longer time to market the interest, and a relatively greater exposure to the risks of the marketplace.

- Pre-IPO private sales of controlling interests should have relatively longer marketing periods than for private sales of unregistered stocks in public companies, because the fact and timing of the IPO event can be uncertain. Furthermore, low pre-IPO stock sales prices may reflect compensation for services rendered. I am not aware of any studies that specifically address discounts observed in sales of controlling interests in pre-IPO companies.
- Private sales of controlling interests in a company that has no expectation of going public should be worth less than an otherwise identical company with an anticipated IPO event. Uncertain or not, an anticipated IPO event should result in a shorter marketing period than not anticipating such an event.
- Pre-IPO sales of non-controlling interests in a company planning an IPO event should be worth less than the controlling interest in the same company even without the planned IPO. The inability to control whether the planned IPO goes forward should result in greater uncertainty and a longer marketing period to liquidate the investment than would be experienced by the controlling investor. Also, low pre-IPO share prices may reflect compensation for services rendered.
- Non-controlling interests in private companies require greater discounts than all of the preceding circumstances because the relative risks of lacking control cause the period of time to liquidate the position to be potentially much longer than for the controlling interest in the same company or for otherwise comparable minority positions in firms with a planned IPO event.

WHY THE EMPIRICAL STUDIES ARE INADEQUATE FOR ESTIMATING DLOM

Unfortunately, the empirical studies of marketability discounts have limited utility to the appraiser opining on the fair market value of a business interest. Several authors have noted that most publicly traded firms do not issue restricted stock. This dearth necessitates samples of limited sizes, in limited industries, with data spread over long periods of time. The result has been substantial standard errors in their estimates.

The restricted stock studies measure the difference in value between a publicly traded stock with and without a time restriction on sale. Left unanswered is whether there is a difference between the restricted stock value of a publicly traded company and the value of that company if it were not publicly traded at all.

The pre-IPO studies reflect substantial standard errors in their estimates for similar reasons, but are also distorted by the facts that the studies necessarily are limited to successful IPOs; there are no post-IPO stock prices for failed IPOs. The discounts observed in the pre-IPO

studies may also reflect uncertainty about whether the IPO event will actually occur, when the IPO event will occur, at what price the event will occur, and compensation for services rendered.

It should also be noted that all of the companies in the restricted stock and pre-IPO studies are, in fact, publicly traded. But essentially none of the privately held companies that are the subject of business valuations have a foreseeable expectation of ever going public. Accordingly, the circumstances of the privately held companies are highly distinguishable from those of the publicly traded companies that are the subjects of the studies. Thus, the pre-IPO studies are of dubious value for determining the DLOM of privately held companies.

Bajaj, et al., studied the difference in value observed when comparing private sales of registered stocks with private sales of unregistered stocks in the same publicly traded company. The result is a measure of the value of registration; it is not a measure of liquidity, much less a measure of DLOM. It is not appropriate to increase the calculated DLOM or otherwise reduce the estimate of FMV for lack of registration. Lack of registration is a factor that is subsumed in the time it takes to market an interest in a private company.[6]

PROBLEMS WITH SOME EXISTING ANALYTICAL METHODS TO MEASURE DLOM

It has been suggested that the Black-Sholes Option Pricing Model ("BSOPM") represents a solution to the DLOM conundrum. It does not. BSCPM is not equivalent to DLOM on a theoretical basis. BSOPM is designed to measure European put and call options. European put options represent the right, but not the obligation, to sell stock for a specified price at a specified point in time. European call options represent the right, but not the obligation, to buy stock for a specified price at a specified point in time. DLOM is not the equivalent of either. Instead, DLOM represents the risk of being unable to sell at any price for a specified period of time.

"At the money" put options have also been suggested as a means of estimating DLOM. Such options represent the right, but not the obligation, to sell stock at the current price at a specified future point in time. Such options do not measure the risk of illiquidity, because the investor is not denied the opportunity to sell for a price that is higher than the put price.

LONGSTAFF PROVIDED THE MEANS TO MEASURE DLOM IN 1995

The critical value difference between publicly traded and privately held companies is that publicly traded investments offer liquidity. All other components of business value are shared: earnings and cash flow, growth, industry risk, size risk, and market risk. However, it is not the value of liquidity per se that DLOM seeks to capture. Instead, it is the ***risk*** associated with ***illiquidity.***

[6] Likewise, brokerage and transactions costs should not be deducted from fair market value appraisals. The result of such deductions would be values that no longer represent the price at which the investments change hands between buyers and sellers – a requirement of fair market value.

But first, what is liquidity? It is the ability to sell quickly when the investor decides to sell. Liquidity allows investors to sell investments quickly to lock in gains or to avoid losses. DLOM, being the result of ***illiquidity*****,** represents the economic risk associated with failing to realize gains or failing to avoid losses on an investment during the period the investor is trying to sell it. This is not necessarily a zero sum game. The value of liquidity (such as observed by Bajaj, et al.) does not translate into the economic risks of faced by investors in private companies. This is because the Bajaj approach does not account for the even longer marketing periods likely to be incurred by investors in private companies compared to investors in unregistered stocks of otherwise publicly traded companies.

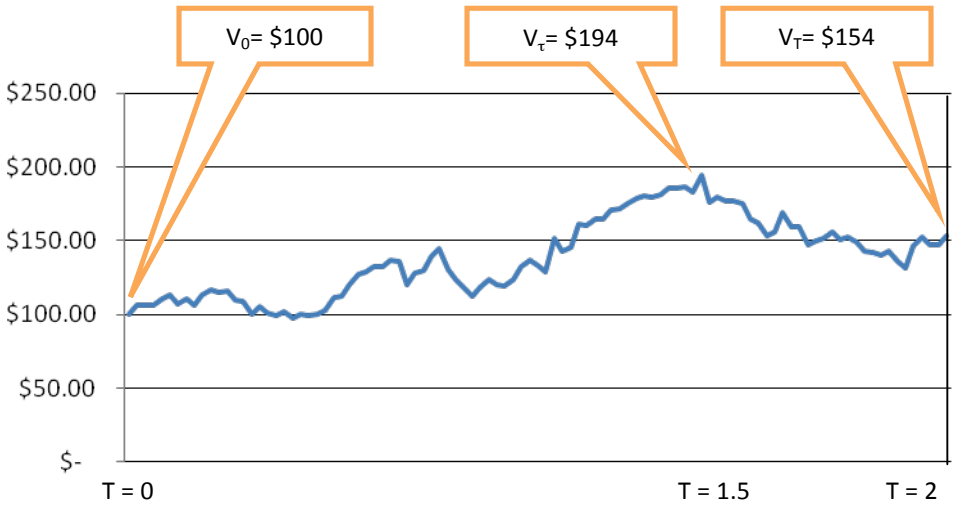
Logically, DLOM can be reduced to price risk faced by an investor during a particular marketing period. In the market for publicly traded stocks, risk reflects the volatility of stock prices. Conversely, investments with no price volatility or that are immediately marketable have no DLOM. Investments with no price volatility can be arbitraged to negate the period of restricted marketing, while volatile investments that are immediately marketable can be sold at the current price to avoid future volatility.

In 1995, UCLA professor Francis A. Longstaff published an article in The Journal of Finance[7] that presented a simple analytical upper bound on the value of marketability using "lookback" option pricing theory. Longstaff's analysis demonstrated that discounts for lack of marketability ("DLOM") can be large even when the illiquidity period is very short. Importantly, the results of Longstaff's formula provide insight into the relationship of DLOM and the length of time of a marketability restriction. Longstaff described the "intuition" behind the results of his formula as follows –

> [Consider] a hypothetical investor with perfect market timing ability who is restricted from selling a security for *T* periods. If the marketability restriction were to be relaxed, the investor could then sell when the price of the security reached its maximum. Thus, if the marketability restriction were relaxed, the incremental cash flow to the investor would essentially be the same as if he swapped the time-*T* value of the security for the maximum price attained by the security. The present value of this lookback or liquidity swap represents the value of marketability for this hypothetical investor, and provides an upper bound for any actual investor with imperfect market timing ability.

[7] The Journal of Finance, Volume I, No. 5, December 1995

Figure 3



For this sample path:

- With restriction, present value of T = 2 at T = 0 is 154*exp(-2*.05) = $139
- Without restriction, could have 194*exp(-1.5*.05) = $180 present value
- Cost of restriction is the difference in present values = $180 - $139 = $41
- DLOM percentage = present value difference divided by investment = 41/100 = 41%

Figure 3 is a graphic presentation of Longstaff's description, in which an investor receives a share of stock worth $100 at time zero, but which he cannot sell for T = 2 years when the stock is worth $154 (present value at T = 0 discounted at a risk free rate of 5% = $139). If at its peak value the stock were worth $194 (present value at T = 0 discounted at a risk free rate of 5% = $180), then the present value cost of the restriction to the investor at T = 0 would be $41, or 41% of his $100 investment. The mathematical formula of this scenario is –

$$D = \left(2 + \frac{\sigma^2 T}{2}\right) N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + \sqrt{\frac{\sigma^2 T}{2\pi}} \exp\left(-\frac{\sigma^2 T}{8}\right) - 1$$

where

σ = volatility

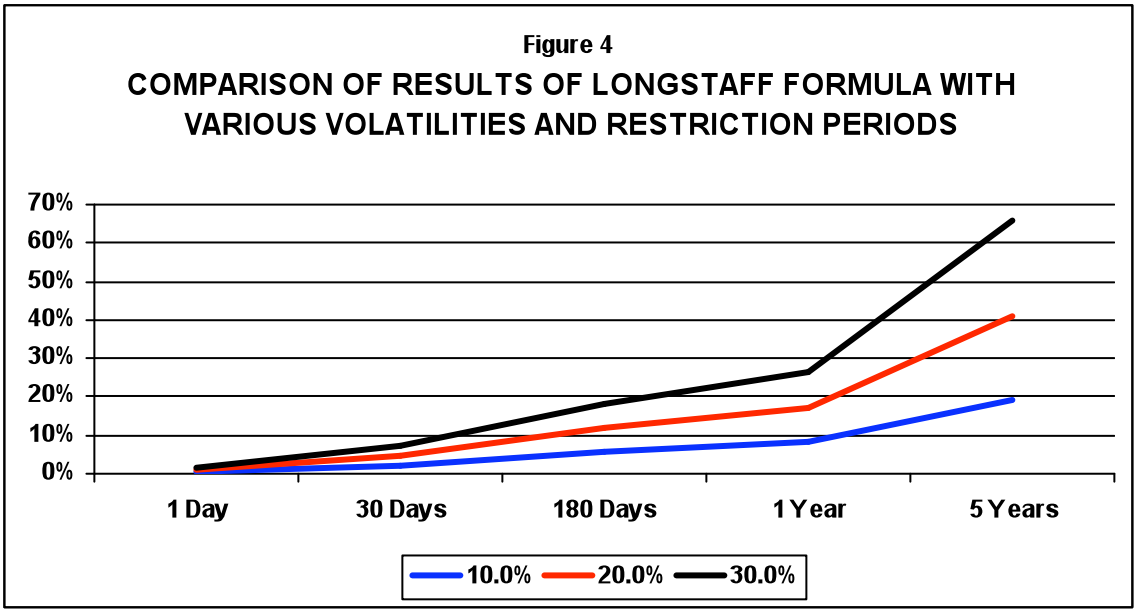
T = Lockout period

$N(\cdot)$ = standard normal cumulative distribution function

The following table presents the results of the formula at various combinations of volatility and length of time of restrictions on marketability.

	Volatility		
	10%	20%	30%
1 Day	0.4%	0.8%	1.3%
30 Days	2.3%	4.7%	7.1%
180 Days	5.8%	11.8%	18.1%
1 Year	8.2%	17.0%	26.3%
5 Years	19.1%	41.0%	65.8%

Figure 4 presents the results graphically:



At VFC, we have been using an application of the Longstaff formula in our business valuations since 2002. Our Longstaff DLOM Methodology calculates a proxy for the subject company's stock price volatility using appropriately selected guideline companies.[8] These are the same companies that VFC uses to apply the publicly traded guideline valuation method in its valuations.[9] We calculate the annualized average stock price volatility for each of the guideline

[8] The use of guideline companies to estimate the subject company's stock price volatility is consistent with the requirements of SFAS 123(R) at paragraph 23 and A22.

[9] I have seen many valuation reports that do not use the publicly traded guideline method and instead rely on some variation of the single period capitalization method. In my opinion, such valuations are fatally flawed. First, single period capitalization can only logically apply to situations in which the expected earnings and cash flow are linear. This circumstance is never true. Second, valuations based solely on single period capitalization – or any one method – lack the checks and balances of using different types of methods (e.g. net assets, public company guideline, private sale transactions, and discounted cash flows methods). Third, discount and capitalization rates of valuations that have not been vetted against guideline values often incorporate substantial and wholly subjective "non-systematic" risk factors that allow the appraiser

companies for an historic period of time equal to the period of time that we believe it will take to market the interest being valued.[10] We then average the calculated volatilities using a simple average.[11] [12] The use of annualized volatility eliminates the "perfect knowledge" and "upper bound" objections by yielding a discount reflective of average knowledge and average volatility.

At VFC, we have applied Longstaff DLOMs to many industries. The following table shows the results from applying it to a hypothetical privately held automobile retailer using the average annualized stock price volatility of five publicly traded automobile retailers[13] as guidelines at two different dates.

COMPARISON OF AUTO INDUSTRY DLOMs USING THE
VFC / LONGSTAFF METHODOLOGY

Marketing Time	14-May-01	31-Dec-05
3 Months	20.6%	11.0%
6 Months	32.6%	15.3%
9 Months	38.1%	20.2%
12 Months	44.1%	22.9%

Figure 5 below graphically presents the results of the table above. As is easily seen, from 2001 to 2005 the automobile dealership industry became less volatile, resulting in reduced DLOMs.

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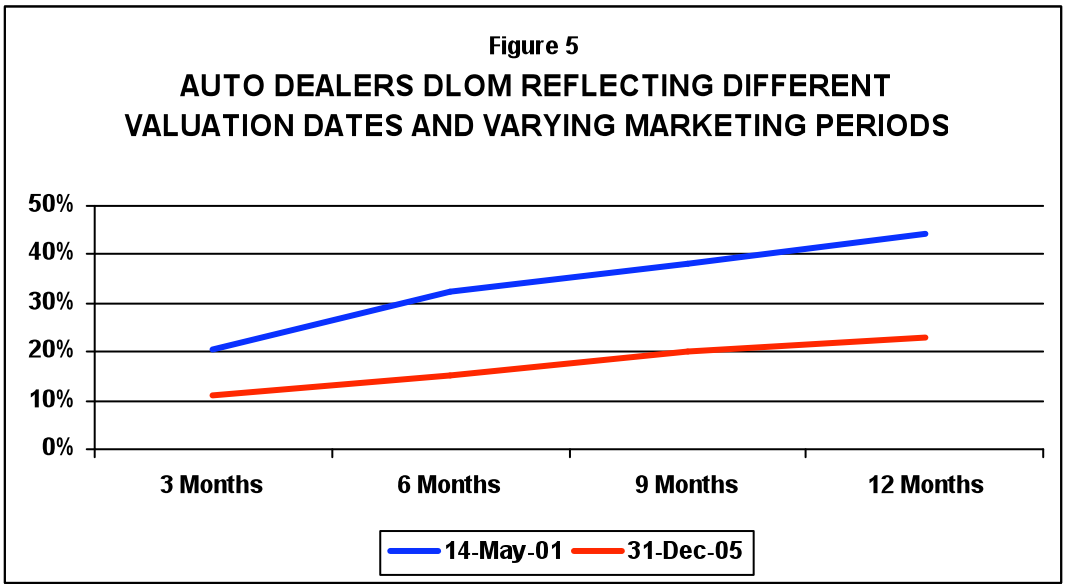
to fabricate his or her answer. In my experience, it is the rare "non-systematic" risk that cannot be quantified and incorporated into the appraiser's projection of future earnings and cash flow, or benchmarked against an appropriately selected guideline company.

[10] Subject to possible adjustment described in SFAS 123(R), using the historical volatility of stock over the most recent time period corresponding in length to the expected period of restriction is consistent with the requirements of the pronouncement. See paragraph A21. Nevertheless, at VFC we are continuing to explore the availability and application of forward looking measures of volatility to the Longstaff DLOM Methodology.

[11] On occasions, we will average the volatilities of the guideline companies using a weighted average that reflects the companies' relative participation in the industry of the subject company.

[12] It is my opinion that harmonic average should not be used for this purpose, or generally when calculating valuation multiples. Harmonic averages are useful when one's goal is to create a portfolio of investments that mirrors the market or a segment of the market. That is not the purpose of a business valuation, which seeks the value of a single particular investment.

[13] AutoNation, Group 1, Lithia, Sonic, and United Auto



APPLYING THE LONGSTAFF METHODOLOGY TO SALES OF LARGE BLOCKS OF PUBLICLY TRADED STOCKS

The Longstaff Methodology provide an effective means of estimating the discount that should be allowed when valuing large blocks of publicly traded stocks. First, estimate the number of shares of stock expected to be sold each day. For example, this may be quantities equal to the SEC Rule 144 "dribble out" rules. Second, use the Longstaff Methodology to compute a separate discount percentage for each day's sales. The discount for the first day's sales may be essentially zero, but the discount for the last day's sales may be very large depending on the historic volatility of the stock and the period of time it will take to liquidate the position. Third, multiply each day's anticipated sales proceeds times one minus the respective day's DLOM percentage. The result is a declining value curve over time as the successively more extended sales lose value as the risks of price volatility increase over time.

CONCLUSION

The Longstaff DLOM Methodology offers a scientific means of reducing the speculation of the appropriate discount for lack of marketability. The methodology is solidly grounded in financial risk theory. The methodology results in a unique, supportable estimate of DLOM for each valuation subject as of the applicable valuation date. The results can be tested and replicated.

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